Maths

1. If $A^2 - A + I = 0$, then the inverse of A is

   1) A + I 2) A 3) A -1 4) I - A

Ans.(4) $A^2 - A + I = 0$

$\Rightarrow A^2 = I - A$

$\Rightarrow A\ A = I - A$

$\therefore A^{-1} = I - A$

2. If the cube roots of unity are I, W, W^2 then the roots of the equation $(x-1)^3+8 = 0$, are

   1) -1, -1+2w, -1 -2w^2 2) -1, -1, -1

   3) -1, 1-2w, 1-2w^2 4) -1, 1+2w, 1+2w^2

Ans.(3) $(x-1)^3 + 8 = 0$

Put x -1 = y

$\rightarrow y^3 + 8 = 0$

$\Rightarrow y = -2, -2\omega, -2\omega^2$

$\Rightarrow x = -1, 1-2\omega, 1-2\omega^2$

3. Let R = {(3, 3) (6, 6) (9, 9) (12, 12), (6, 12) (3, 9) (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. The relation is

   1) reflexive and transitive is 2) reflexive only

   3) an equivalence relation 4) reflexive and symmetric only

Ans.(1) R is reflexive and transitive but not symmetric because $(9, 3) \notin R$

4. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   1) 2ba 2) ab 3) $\sqrt{ab}$ 4) $\frac{a}{b}$

Ans.(1) Area of greatest rectangle

$= \frac{1}{2} \times 4ab$

= 2ab

5. The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where c > 0, is a parameter, is of order and degree as follows :

   1) order 1, degree 2 2) order 1, degree1

   3) order 1, degree 3 4) order 2, degree 2

Ans.(3) $y^2 = 2c(x + \sqrt{c}\ )$ c < 0

since there is only one arbitary constant

order = 1

$2y\frac{dy}{dx} = 2c$

$\Rightarrow c = y\frac{dy}{dx}$

$\Rightarrow y^2 = 2\left(y\frac{dy}{dx}\right)x + 2\left(y\frac{dy}{dx}\right)^{3/2}$

Elimainating radical powers,

We get, degree = 3

6. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+...+\frac{1}{n}\sec^2 1\right]$ is

1) $\frac{1}{2}\sec 1$ 2) $\frac{1}{2}\text{cosec}\,1$ 3) tan 1 4) $\frac{1}{2}\tan 1$

Ans.(4) $\frac{1}{2}\tan 1$ $\underset{n\to\infty}{\text{Lt}}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}.\sec^2\frac{4}{n^2}+.....+\frac{n}{n^2}\sec^2\frac{n^2}{n^2}\right]$

$\Rightarrow \underset{n\to\infty}{\text{Lt}}\ \frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2$

$\Rightarrow \int_0^1 x\sec^2(x^2)dn$

$\Rightarrow \int_0^1 \frac{1}{2}\sec^2 t\, dt$ let $x^2 = t$

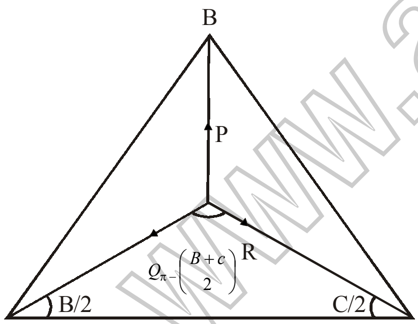
$2x\ dx = dt$

$\Rightarrow \frac{1}{2}\tan 1$

7. ABC is a triangle. Forces $\vec{P},\vec{Q},\vec{R}$ acting along IA, IB and IC respectively are in equilibrium, where I is the incentre of ΔABC. Then P : Q : R is

1) sin A : sin B : sin c
2) $\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}$
3) $\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$
4) cos A : cos B : cos C

Ans.(3)



Applying Lami's theorem

$$\frac{P}{\sin\left(\frac{B+C}{2}\right)}=\frac{Q}{\sin\left(\frac{A+C}{2}\right)}=\frac{R}{\sin\left(\frac{A+B}{2}\right)}$$

$\Rightarrow P:Q:R=\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$

8. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

1) 22.0 2) 20.5 3) 25.5 4) 24.0

Ans.(4) Mode = 3 median - 2 mean

= 66 - 42

= 24

9. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point PQ is

1) $y^2 - 4x + 2 = 0$ 2) $y^2 + 4x + 2 = 0$ 3) $x^2 + 4y + 2 = 0$ 4) $x^2 - 4y + 2 = 0$

Ans.(1) Let the mid point be = (h, k)

Let Q = $(x_1\ y_1)$

$$\frac{1+x_1}{2} = h \quad \frac{0+y_1}{2} = k$$

$$\Rightarrow x_1 = 2h - 1$$

$$y_1 = 2k$$

$x_1\ y_1$ lies on $y^2 = 8x$

$$\Rightarrow 4k^2 = 8(2h-1)$$

$$\Rightarrow k^2 = 2(2h-1)$$

$\therefore$ The eqn. of locus is

$$y^2 - 4x + 2 = 0$$

10. If C is the mid point of AB and P is any point outside AB, then

1) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$ 2) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

3) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$ 4) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$

Ans.(1) 

$$\overrightarrow{PA} = \overrightarrow{PC} + \overrightarrow{CA} \quad ..(1)$$

$$\overrightarrow{PA} = \overrightarrow{PC} + \overrightarrow{BC} \quad ...(2)$$

Adding $2\overrightarrow{PA} = 2\overrightarrow{PC} + \overrightarrow{CA} + \overrightarrow{BC}$

$$\Rightarrow \overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$$

11. If the coeffieients of rth, (r + 1)th and (r+2)th terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation

1) $m^2 - m(4r-1) + 4r^2 - 2 = 0$ 2) $m^2 - m(4r+1) + 4r^2 + 2 = 0$

3) $m^2 - m(4r+1) + 4r^2 - 2 = 0$ 4) $m^2 - m(4r-1) + 4r^2 + 2 = 0$

Ans.(3) Coefficients of r, $(r+1)^{th}$ or $(r+2)^{th}$ terms are in A.P.

$$\Rightarrow 2\ {}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$$

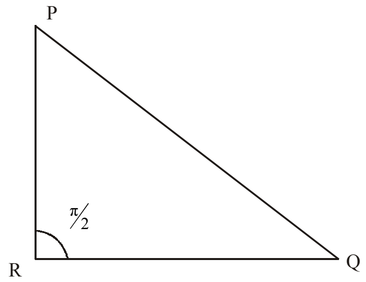
$$\Rightarrow \frac{2}{r(m-r)} = \frac{1}{(m-r+1)(m-r)} + \frac{1}{(r+1)r}$$

$$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$$

12. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0, a \neq 0$ then

1) a = b + c 2) c = a + b 3) b = c 4) b = a + c

Ans.(2)



$\Rightarrow P + Q = \pi/2$

$\Rightarrow \dfrac{P+Q}{2} = \pi/4$

$\Rightarrow \tan\left(\dfrac{P}{2} + \dfrac{Q}{2}\right) = 1$

$\dfrac{\tan P/2 + \tan Q/2}{1 - \tan P/2 \tan Q/2} = 1$

$\therefore \tan P/2$ & $\tan Q/2$ are the root of the eqn. $ax^2 + bx + c = 0$

$\Rightarrow \dfrac{-b/a}{1 - c/a} = 1 \Rightarrow -\dfrac{b}{a} = 1 - c/a$

$\Rightarrow -b = a - c$

$\Rightarrow c = a + b$

13. The systems of equations

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

has no solution, if α is

1) -2
2) either -2 or 1
3) not -2
4) 1

Ans.(1) $\Delta = 0$

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

From options, it can be easily verified that $\alpha = -2$

14. The value of α for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is

1) 1
2) 0
3) 3
4) 2

Ans.(1) Let α, β be the roots

$\alpha + \beta = +(a - 2) = a - 2$

$\alpha \beta = -(a + 1)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a-2)^2 + 2(a+1)$

$= a^2 - 4a + 4 + 2a + 2$

$= a^2 - 2a + 6$

$= (a-1)^2 + 5$

$\therefore \alpha^2 + \beta^2$ is least value a = 1

15. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals

1) -2 2) 3 3) 2 4) 1

Ans.(4) Let α, $\alpha + 1$ be the roots

$\alpha + \alpha + 1 = b$

$\Rightarrow 2\alpha + 1 = b$

$\alpha(\alpha + 1) = c$

$b^2 - 4c$

$= (2\alpha + 1)^2 - 4(\alpha^2 + \alpha)$

$= 4\alpha^2 + 4\alpha + 1 - (4\alpha^2 + 4\alpha) = 1$