16. If the letters of the words SACHIN are arrange in all possible ways and these words are written out as in dictionary, then the words are SACHIN appears at serial number.

1) 601 2) 600 3) 603 4) 602

Ans.(1) S A C H I N

A - order : - A C H I N S

Rank = (5!) + 0 (4!) + 0(3!)

+ 0 (2!) + 0(1!) + 1

$= 5\times120+1$

= 601

17. The value of ${}^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$ is

1) ${}^{55}C_4$ 2) ${}^{55}C_3$ 3) ${}^{56}C_3$ 4) ${}^{56}C_4$

Ans.(4) ${}^{50}C_4+\sum_{r=1}^{6}$ 56 - rc_3

${}^{50}C_4+{}^{55}C_3+{}^{54}C_3+{}^{53}C_3+{}^{52}C_3+{}^{51}C_3+{}^{50}C_3$

$= {}^{56}C_4$

18. If $A=\begin{bmatrix}1&0\\1&1\end{bmatrix}$ and $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, then which of the following holds for all $n\geq1$, by principle of mathematical induction

1) $A^n = nA-(n-1)I$

2) $A^2=2^{n-1}A-(n-1)I$

3) $A^n=nA+(n-1)I$

4) $A^n=2^{n-1}A+(n-1)I$

Ans.(1) $A=\begin{pmatrix}1&0\\1&1\end{pmatrix}$

$I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

$A^2=\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}$

$=\begin{pmatrix}1+0&0+0\\1+1&0+1\end{pmatrix}$

$=\begin{pmatrix}1&0\\2&1\end{pmatrix}$

$A^3=\begin{pmatrix}1&0\\2&1\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}=\begin{pmatrix}1&0\\3&1\end{pmatrix}$

$A^2=2A-I$

$=\begin{pmatrix}2&0\\2&2\end{pmatrix}-\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}1&0\\2&1\end{pmatrix}$

$A^3=3A-2I$

$=\begin{pmatrix}3&0\\3&3\end{pmatrix}-\begin{pmatrix}2&0\\0&2\end{pmatrix}=\begin{pmatrix}1&0\\3&1\end{pmatrix}$

19. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$ then a and b satisfy the relation

1) a - b = 1   2) a + b = 1   3) $\frac{a}{b}=1$ 4) ab = 1

Ans.(4) $\left(x^2+\frac{1}{bx}\right)^{11}$

$r=\frac{2\times 11-7}{3}=5$

coefficient of $x^7={}^{11}C_5\,a^6\,\frac{1}{b^5}$

$\left(ax-\frac{1}{bx^2}\right)^{11}$

$r=\frac{11+7}{3}=6$

Coefficeint of $x^{-7}={}^{11}C_6\,a^5\,\frac{1}{b^6}$

${}^{11}C_6\,a^6\,\frac{1}{b^5}={}^{11}C_6\,a^5\,\frac{1}{b^6}$

$\Rightarrow ab=1$

20. Let f : (-1, 1) $\rightarrow$ B, be a function defined by f(x) = $\tan^{-1}\frac{2x}{1-x^2}$ then f is both one-one and onto when B is the interval

1) $\left(0,\frac{\pi}{2}\right)$ 2) $\left[0,\frac{\pi}{2}\right]$ 3) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ 4) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Ans.(4) Since $f\rightarrow(-1,1)\rightarrow B$ &

$f(x)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

if is both one - one & onto i , e (bijective)

$\Rightarrow$ if (x) inverse tangent of n whose Range is always $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

21. If z_1 and z_2 are two non-zero comple numbers such that $|z_1+z_2|=|z_1|+|z_2|$, then arg z_1 - arg z_2 is equal to

1) $\frac{\pi}{2}$ 2) $-\pi$ 3) 0   4) $\frac{-\pi}{2}$

Ans.(3) $|z_1+z_2|^2=[|z_1|+|z_2|]^2$

$\Rightarrow |z_1|^2+|z_2|^2+2|z_1|\,|z_2|\cos(\theta_1-\theta_2)$

$=|z_1|^2+|z_2|^2+2|z_1|\,|z_2|$

$\cos\,(\theta_1-\theta_2)=1$

$\Rightarrow \theta_1-\theta_2=0$

$\Rightarrow \mathrm{Arg}(z_1)-\mathrm{Arg}(z_2)=0$

22. If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$, then z lies on

1) an ellipse 2) a circle 3) a straight line 4) a parabola

Ans.(3) $w = \frac{z}{z - \frac{1}{3}i}$

$$|w| = \frac{|z|}{\left|z - \frac{1}{3}i\right|}$$

$$\Rightarrow \left|z - \frac{1}{3}i\right| = |z|$$

$\Rightarrow$ z lies on the $\perp$r bisector of the line segment joining $\left(0, \frac{1}{3}\right)$ and $(0, 0)$

23. If $a^2 + b^2 + c^2 = -2$ and

$$f(x) = \begin{vmatrix} 1 + a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

then $f(x)$ is a polynomial of degree

1) 1 2) 0 3) 3 4) 2

Ans.(4) $a^2 + b^2 + c^2 = -2$

In $f(x), c_1 \Rightarrow c_1 + c_2 + c_3$

$$\Rightarrow f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2)x+2x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2)x+2x & 1+b^2x & (1+c^2)x \\ 1+(a^2+b^2+c^2)x+2x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 0 & x-1 & 0 \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$\Rightarrow -(x-1)\left[1 + c^2x - x - c^2x\right]$$

$$\Rightarrow (1-x)^2 \Rightarrow \text{degree} = 2$$

24. The normal to the curve $x = \alpha\,(\cos\theta + \theta\sin\theta),\ y = a\,(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that

1) It passes through the origin

2) It makes angle $\frac{\pi}{2} + \theta$ with the x - axis

3) It passes through $\left(a\frac{\pi}{2}, -a\right)$

4) It is at a constant distance from the origin

Ans.(2 , 4) $x = a(\cos\theta + \theta\sin\theta)$

$y = a(\sin\theta - \theta\cos\theta)$

$$\frac{dy}{dx} = \frac{a\,(\cos\theta + \theta\sin\theta - \cos\theta}{a\,(-\sin\theta + \theta\cos\theta + \sin\theta} = \frac{\sin\theta}{\cos\theta}$$

$\therefore$ slope of the normal

$$\Rightarrow -\frac{dx}{dy} = -\frac{\cos\theta}{\sin\theta}$$

$\Rightarrow$ Slope of the normal is $\frac{\pi}{2} + \theta$

$\therefore$ Eqn. of the normal

$$\Rightarrow y - a\sin\theta + a\,\theta\cos\theta = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\,\theta\sin\theta)$$

$$\Rightarrow y\sin\theta - a\sin^2\theta + a\,\theta\cos\theta\sin\theta = -\,x\cos\theta + a\cos^2\theta + a\,\theta\cos\theta\sin\theta$$

$$\Rightarrow x\cos\theta + y\sin\theta = a$$

$\Rightarrow$ It is at a constant distance from the origin.

25. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

Interval	Function
1) $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
2) $[2, \infty)$	$2x^3 - 3x^2 + 12x + 6$
3) $\left(-\infty, \frac{1}{3}\right]$	$3x^3 - 2x^2 + 1$
4) $(-\infty, -4]$	$x^3 - 6x^2 + 6$

Ans.(3)

$f(x) = 3x^3 - 2x^2 + 1$

$f'(x) = 6x - 2 \geq 0 \Rightarrow x \geq 1/3$

Option (3) is incorrect. Checking other function similarly we find that they are correctly matched.

26. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then

$\lim\limits_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - a)^2}$ is euql to

1) $\frac{a^2}{2}(\alpha - \beta)^2$

2) 0

3) $\frac{-a^2}{2}(\alpha - \beta)^2$

4) $\frac{1}{2}(\alpha - \beta)^2$

Ans.(1) We have $\lim\limits_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$

$$= 2\lim_{x \to \alpha} \frac{\sin^2\left\{\frac{(ax^2 + bx + c}{2}\right\}}{(x - \alpha)^2} = 2\lim_{x \to \alpha} \frac{\sin^2\left\{\frac{a(x - \alpha)\,(x - \beta)}{2}\right\}}{(x - \alpha)^2}$$

$$\left[\begin{array}{l} \because \alpha, \beta \text{ are roots of } ax^2 + bx + c = 0 \\ \therefore ax^2 + bx + c = a(x - \alpha)(x - \beta) \end{array}\right]$$

$$= 2\lim_{x \to a} \left[\frac{\sin \frac{a(x - \alpha)(x - \beta)}{2}}{\frac{a(x - \alpha)(x - \beta)}{2}}\right]^2 . \frac{a^2}{4}(x - \beta)^2$$

$$= 2(1)^2 \frac{a^2}{4}(\alpha - \beta)^2 = \frac{a^2}{2}(\alpha - \beta)^2$$

27. Suppose f(x) is differentiable at x = 1 and $\lim\limits_{h \to 0} \frac{1}{h} f(1 + h) = 5$ then f(1) equals

1) 3 2) 4 3) 5 4) 6

Ans.(4) Applying L' Hospital rule

$\mathop{Lim}\limits_{h \to 0} \frac{f'(1 + h)}{1} = 5$

$\Rightarrow f'(1) = 5$

28. Let f be differentiable for all x. If f(1) = -2 and $f'(x) \geq 2$ for x $\in$ [1, 6], then

1) f(6) $\geq$ 8 2) f(6) < 8

3) f(6) < 5 4) f(6) = 5

Ans.(2) Here from LMVT

$\frac{f(6) - f(1)}{5} \geq 2$

$f(6) \geq 8.$

29. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \le (x-y)^2, x, y \in$ and f(0) = 0, then f(1) equals

1) -1   2) 0   3) 2   4) 1

Ans.(2) $\lim_{x \to y} \left| \frac{f(x) - f(y)}{x - y} \right| \le \lim_{x \to y} \left| x - y \right|$

or $|f'(x)| \le 0$

$\Rightarrow f'(x) = 0$

$\Rightarrow f(x)$ is constant, As $\Rightarrow f(0) = 0$

$\therefore f(1) = 0$

30. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{\frac{3}{2}} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

1) $1 - \frac{3}{8}x^2$

2) $3x + \frac{3}{8}x^2$

3) $-\frac{3}{8}x^2$

4) $\frac{x}{2} - \frac{3}{8}x^2$

Ans.(3) $\frac{(1+x)^{3/2} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{1/2}}$

$$= \frac{1 + \frac{3}{2}x + \frac{\frac{3}{2}\cdot\frac{1}{2}}{2}x^2 - \left(1 + \frac{3x}{2} + \frac{3.2}{2}\frac{x^2}{4}\right)}{(1-x)^{1/2}}$$

$$= \frac{-\frac{3}{8}x^2}{(1-x)^{1/2}} = -\frac{3}{8}x^2(1-x)^{-1/2}$$

$$= -\frac{3}{8}x^2\left(1 + \frac{x}{2} + \ldots\right) = -\frac{3}{8}x^2$$