31. If $x=\sum_{n=0}^{\infty} a^n, y=\sum_{n=0}^{\infty} b^n, z=\sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P and $|a|<1, |b|<1, |c|<1$ then x, y, z are in

1) GP
2) AP
3) Arithmetic - Geometric Progression
4) HP

Ans.(4) $x=\frac{1}{1-a} \Big| y=\frac{1}{1-b} \Big| z=\frac{1}{1-c}$

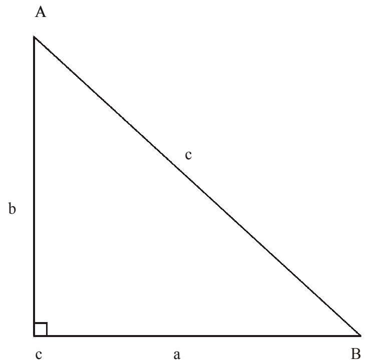
a, b, c, are in A.P

$\therefore \frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H -P

$\Rightarrow x, y, z$ are in H-P

32. In a triangle ABC, let $\angle C=\frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then 2(r+R) equials

1) b + c
2) a + b
3) a + b + c
4) c + a

Ans.(2) 

Let a = b = 1

$\Rightarrow c=\sqrt{2}$

r = (s-c) tan c/2

$=\left(\frac{2+\sqrt{2}}{2}-\sqrt{2}\right)\tan \Pi/4$

$\Rightarrow \frac{2+\sqrt{2}-2\sqrt{2}}{2}$

$r \Rightarrow \frac{2-\sqrt{2}}{2}=1-\frac{1}{\sqrt{2}}$

$R=\frac{a\,b\,c}{4\,\Delta}=\frac{1\times 1\times \sqrt{2}}{4\times \frac{1}{2}\times 1\times 1}$

$=\frac{1}{\sqrt{2}}$

$\therefore 2\,(r+R)=2\left(1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)=2$

$=a+b$

33. If $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

1) $2\sin 2\alpha$
2) 4
3) $4\sin^2\alpha$
4) $-4\sin^2\alpha$

Ans.(3) $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$

$$\Rightarrow \cos^{-1}\left(\frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}\right) = \alpha$$

$$\Rightarrow \frac{xy}{2} + \frac{\sqrt{1-x^2}\sqrt{4-y^2}}{2} = \cos\alpha$$

$$\Rightarrow (1-x^2)(4-y^2) = (2\cos\alpha - xy)^2$$

$$4 - 4x^2 - y^2 + x^2y^2 = 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$$

$$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4 - 4\cos^2\alpha = 4\sin^2\alpha$$

34. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in

1) G.P.
2) A.P.
3) Arithmetic - Geometric - Progression
4) H . P.

Ans.(2) In a ΔABC,

Let the altitudes be p_1, p_2, p_3 from the vertices A, B, C respectively then

$$P_1 = \frac{2\Delta}{a}, P_2 = \frac{2\Delta}{b}, P_3 = \frac{2\Delta}{c}$$

$\Rightarrow \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in HP

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in HP

$\Rightarrow$ a, b, c are in AP

$\Rightarrow$ 2 R sin A, 2 R sin B, 2 R sin C are in AP

$\Rightarrow$ sin A, Sin B, sin C in AP.

35. If $I_1 = \int_0^1 2^{x^2}\,dx, I_2 = \int_0^1 2^{x^3}\,dx,$

$I_3 = \int_1^2 2^{x^2}\,dx$ and $I_4 = \int_1^2 2^{x^3}\,dx,$ then

1) $I_2 > I_1$
2) $I_1 > I_2$
3) $I_3 = I_4$
4) $I_3 > I_4$

Ans.(2) $I_1 = \int_0^1 2^{x^2}\,dx,$ $\qquad I_2 = \int_0^1 2^{x^3}\,dx$

$I_3 = \int_0^1 2^{x^2}\,dx,$ $\qquad I_4 = \int_0^2 2^{x^3}\,dx$

For $x \in (0, 1)$

$x^2 > x^3$

$\Rightarrow 2^{x^2} > 2^{x^3}$

$\Rightarrow \int_0^1 2^{2^2}\ dx > \int_0^1 2^{x^3}\ dx$

$= I_1 > I_2$

For $x \in (1,\ 2)$

$x^3 > x^2$

$2^{x^3} > 2^{x^2}$

$\int_1^2 2^{x^3}\ dx > \int_1^2 2^{x^2}\ dx$

$I_4 > I_3$

36. The area enclosed between the curve y = $\log_e$ (x+e) and the coordinate axes is

1) 1 2) 2 3) 3 4) 4

Ans.(1) Required area

$= \int_{1-e}^{0} \log(x+e)\, dx$

Put x + e = t

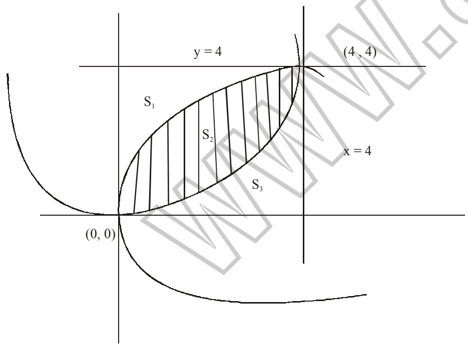
$= \int_{1}^{e} \log t\ dt$

$= [x \log x - x]_1^e = 1$ sq. units

37. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4, y = 4 and the coordinate axes. If S_1, S_2, S_3 are respectively the area of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

1) 1 : 2 : 1 2) 1 : 2 : 1 3) 2: 1 : 2 4) 1: 1 : 1

Ans.(4)



$S_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$

$= 16/3$

$\therefore S_2 + S_2 + S_3 = 4 \times 4 = 16$

$\therefore S_2 + S_3 = 16 - \frac{16}{3} = \frac{32}{3}$

$\Rightarrow S_1 = {}^{16}\!/_{3}, = S_3\ {}^{16}\!/_{3}$

$\therefore S_1 : S_2 : S_3 = 1:1:1$

38. If $x\frac{dy}{dx} = y$ (log y - log x + 1), then the solution of the equation is

1) $y\log\left(\frac{x}{y}\right) = cx$ 2) $x\log\left(\frac{y}{x}\right) = cy$

3) $\log\left(\frac{y}{x}\right) = cx$ 4) $\log\left(\frac{x}{y}\right) = cy$

Ans.(3) If $x\frac{dy}{dx} = y\,(\log y - \log x + 1)$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$$

Let y = vx

$$\Rightarrow v + x\frac{dv}{dx} = V\,(\log v + 1)$$

$$\Rightarrow v + x\frac{dv}{dx} = V\,(\log v + v)$$

$$\Rightarrow \int \frac{dv}{v\log v} = \int \frac{dx}{x}$$

$$\Rightarrow \log(\log v) = \log x + \log c$$

log (log v) = log (cx)

$$\Rightarrow \log {}^{y}\!/\!_{x} = c\,x$$

39. The line parallel to the x-axis and passing through the intersection of the lines ax + 2by + 3b = 0 and bx - 2ay - 3a= 0 where (a, b) $\neq$ (0, 0) is

1) below the x-axis at a distance of $\frac{3}{2}$ from it

2) below the x-axis at a distance of $\frac{2}{3}$ from it

3) above the x-axis at a distance of $\frac{3}{2}$ from it

4) above the x-axis at a distance of $\frac{2}{3}$ form it

Ans.(1) ax + 2by = -3 b

bx - 2ay = 3a

Solving these two equations we get y = $-\frac{3}{2}$

40. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

1) $\frac{1}{36\pi}$ cm/min. 2) $\frac{1}{18\pi}$ cm/min. 3) $\frac{1}{54\pi}$ cm/min. 4) $\frac{5}{6\pi}$ cm/min.

Ans.(2) $V = \frac{4}{3}\pi(x+10)^3$ Where x is thickness of ice.

$$\therefore \frac{dV}{dt} = 4\pi(10+x)^2\frac{dx}{dt}$$

$$\therefore \left(\frac{dx}{dt}\right)\text{at } x = 5; \quad = \frac{1}{18\pi}\text{ cm/min.}$$

41. $\int \left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to

1) $\frac{\log x}{(\log x)^2+1}+C$

2) $\frac{x}{x^2+1}+C$

3) $\frac{xe^x}{1+x^2}+C$

4) $\frac{x}{(\log x)^2+1}+C$

Ans.(4) $Let\ f(x)\int\left\{\frac{\log x-1}{1+(\log x)^2}\right\}^2 dx$

Differentiating from the optins Q(x)

$\frac{d}{dx}\left[\frac{x}{(\log x)^2+1}+c\right]$

$\Rightarrow \frac{\left[(\log x)^2+1\right]-x\left[(2\log x)\frac{1}{x}\right]}{\left[(\log x)^2+1\right]}$

$\Rightarrow \frac{(\log x-1)^2}{\left[(\log x)^2+1\right]^2}$

42. Let f : $R \to R$ be a differentiable function having f(2) = 6, $f'(2)=\left(\frac{1}{48}\right)$. Then $\underset{x\to 2}{Lt}\int_{6}^{f(x)}\frac{4t^3}{x-2}dt$ equals

1) 24 2) 36 3) 12 4) 18

Ans.(4) $\lim_{x\to 2}\frac{\int_{6}^{f(x)}4t^3\,dt}{x-2}$ (0/0 form)

$=\lim_{x\to 2}\frac{4(f(x))^3\times f'(x)}{1}=4(f(2))^3\times f'(2)=18$

43. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve y = $f(x)$, x-axis and the ordinates $x=\frac{\pi}{4}$ and $x=\beta>\frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\,\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

1) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$

2) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

3) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$

4) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

Ans.(4) $\int_{\pi/4}^{\beta} f(x)dx=\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta$

$\therefore f'(\beta)=\sin\beta+\beta\cos\beta-\frac{\pi}{4}\cos\beta+\sqrt{2}$

$f\left(\frac{\pi}{2}\right)=1-\frac{\pi}{4}+\sqrt{2}$

44. The locus of a point P (α, β) moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

1) an ellipse 2) a circle 3) a parabola 4) a hyperbola

Ans.(4) $\Rightarrow \beta^2 = a^2 \alpha^2 - b^2$

$\therefore$ the locus is $a^2 x^2 - y^2 = b^2$

$$\Rightarrow \frac{x^2}{\frac{1}{a^2}} - \frac{y^2}{1} = b^2$$

$\therefore$ the locus is a- hyperbola

45. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda} z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ the value of λ is

1) $\frac{5}{3}$ 2) $\frac{-3}{5}$

3) $\frac{3}{4}$ 4) $\frac{-4}{3}$

Ans.(1) $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-1}{2}$ $(a_1, b_1, c_1) = (1, 2, 2)$

$2x - y + \sqrt{\lambda}\, z + 4 = 0$

$(a_2, b_2, c_2) = (2, -1, \sqrt{\lambda})$

$$\sin\theta = \frac{2 - 2 + 2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$$

$$\Rightarrow \frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$$

$\Rightarrow 5 + \lambda = 4\lambda$

$\Rightarrow \lambda = 5/3$