46. The angle between the lines 2x = 3y = -z and 6x = - y = - 4z is

1) 0^0 2) 90^0 3) 45^0 4) 30^0

Ans.(2) $2x = 3y = -z,$

$$\Rightarrow \frac{x}{1\backslash 2} = \frac{y}{1/3} = \frac{z}{-1}$$

$$(a_1, b_1, c_1) = \left(\frac{1}{2}, \frac{1}{3}, -1\right)$$

6x = -y = -4z

$$\Rightarrow \frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{-1/4}$$

$$(a_2\ b_2\ c_2) = \left(\frac{1}{6}, -1, -\frac{1}{4}\right)$$

$$a_1 a_2 + b_1 b_2 + c_1 c_2$$

$$= \frac{1}{12} - \frac{1}{3} + \frac{1}{4}$$

$$= 0$$

47. If the plane 2ax - 3ay +4az + 6 = 0 passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8x - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then α equals

1) -1 2) 1 3) -2 4) 2

Ans.(3) Centre of $S_1 = (-3, 4, 1)$

Centre $S_2 = (5, -2, 1)$

Mid pt. of S_1S_2 (1, 1, 1)

2ax - 3ay + 4az + 6 = 0

passes through (1, 1, 1)

2a- 3a + 4a = -6

$\Rightarrow 3a = -6$

$\Rightarrow a = -2$

48. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is

1) $\frac{10}{9}$ 2) $\frac{10}{3\sqrt{3}}$ 3) $\frac{3}{10}$ 4) $\frac{10}{3}$

Ans.(2) $\vec{r} = 2i - 2j + 3k + \lambda(i - j + 4k)$

$\vec{r}.(i + 5j + k) = 5$

the line passes through (2, -2, 3)

$\therefore$ the reqd distance = $\left|\frac{2 - 10 + 3 - 5}{\sqrt{1 + 25 + 1}}\right|$

$$= \frac{10}{\sqrt{27}}$$

$$= \frac{10}{3\sqrt{3}}$$

49. For any vector $\vec{a}$, the value $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to

1) $3\vec{a}^2$
2) $\vec{a}^2$
3) $2\vec{a}^2$
4) $4\vec{a}^2$

Ans.(3) Let $\vec{a} = xi + yj + zk$

$\vec{a}\times\hat{i} = (xi+yj+zk)\times i$

$= -y\,k + z\,j$

$(\vec{a}\times\hat{i})^2 = y^2+z^2$

|| ly $(\vec{a}\times\hat{i})^2 = z^2+x^2$

$(\vec{a}\times k)^2 = x^2+y^2$

$\therefore$ The required sum

$= 2(x^2+y^2+z^2)$

$= 2\vec{a}^2$

50. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is

1) (-1, 2)
2) (-1, -2)
3) (1, -2)
4) $\left(1,-\frac{1}{2}\right)$

Ans.(3) Let a, b and c be 1, $\frac{1}{2},\frac{1}{3}$

$\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$

$\Rightarrow x + 2y + 3 = 0$

(1, -2) satisfies the equation

51. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (-1, 2) and (3, 2), then the centroid of the traingle is

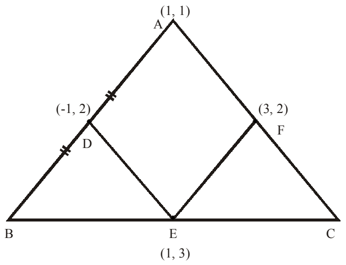
1) $\left(-1,\frac{7}{3}\right)$

2) $\left(\frac{-1}{3},\frac{7}{3}\right)$

3) $\left(1,\frac{7}{3}\right)$

4) $\left(\frac{1}{3},\frac{7}{3}\right)$

Ans.(3)



E = -1 + 3 -1, 2+ 2 -1

= (1, 3)

Centried of Δ ABC = centroid of Δ DEF

$$=\left(\frac{-1+1+3}{3},\frac{2+2+3}{3}\right)$$

$$=\left(1,\frac{7}{3}\right)$$

52. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line 5x + by - a = 0 passes through P and Q for

1) exactly one value of α
2) no value of α
3) infinitely many values of α
4) exactly two values of α

Ans.(2) Radical axis is

5a x + (c-d) y + (a+1) = 0 ....(1)

5 x + by - a = 0 ....(2)

$$\frac{5a}{5}=\frac{a+1}{-a}$$

$$\Rightarrow -a^2=a+1$$

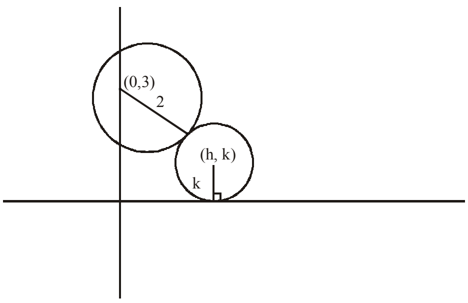
$$\Rightarrow a^2+a+1=0$$

No real value of a satisfies

53. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is

1) an ellipse
2) a circle
3) a hyperbola
4) a parabola

Ans.(4)



$$(x-h)^2+(y-k)^2=k^2$$

$$(x-0)^2+(y-3)^2=4$$

$$d[(0,3),(h,k)]=2+k$$

$\Rightarrow \sqrt{h^2 + (k-3)^2} = (k+2)$

$\Rightarrow h^2 + (k-3)^2 = (k+2)^2$

$\Rightarrow h^2 + k^2 + 9 - 6k = k^2 + 4k + 4$

$\Rightarrow h^2 - 10k + 5 = 0$

$\therefore$ locus of its centre is

$h^2 + 10k + 5 = 0$

i.e., $x^2 + 10y + 5 = 0$

$x^2 = -10y - 5$

$x^2 = -5(2y+1)$

$\therefore$ The locus is a parabola

54. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is

1) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$

2) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

3) $x^2 + y^2 - 2ax - 3by(a^2 - b^2 - p^2) = 0$

4) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

Ans.(4) It the equation be

$x^2 + y^2 + 2gx + 2fy + c = 0$

passes through (a, b)

$a^2 + b^2 + 2ag + 2bf + c = 0$ ....(1)

It cuts $x^2 + y^2 = p^2$ orthogonally

2(g.0 + f.0) = c-p^2

$\Rightarrow c - p^2 = 0$

$c = p^2$

Substituting the value of c in 1
we get

$a^2 + b^2 + 2ag + 2bf + p^2 = 0$

$\Rightarrow 2ag + 2bf + a^2 + b^2 + p^2 = 0$

locus of its centre (-g, -f) is

$-2ag - 2bf + a^2 + b^2 + p^2 = 0$

$\Rightarrow 2ag + 2bf - (a^2 + b^2 + p^2) = 0$

$2ax + 2by - (a^2 + b^2 + P^2) = 0$

55. An ellipse has *OB* as semi minor axis, F and F' its focil and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

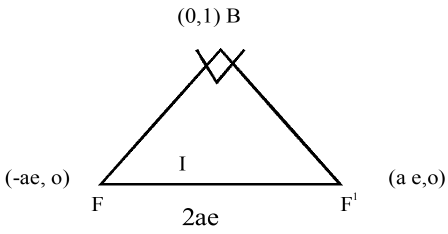
1) $\frac{1}{\sqrt{2}}$

2) $\frac{1}{2}$

3) $\frac{1}{4}$

4) $\frac{1}{\sqrt{3}}$

Ans.(1)



Slope of F B = $\frac{b}{ae}$

Slope of $F^1B = \frac{b}{-ae}$

$\frac{b}{ae} \times - \frac{b}{ae} = -1$

$\Rightarrow b^2 = a^2e^2$

$b^2 = a^2 - a^2e^2$

$2a^2e^2 = a^2$

$\Rightarrow e^2 = \frac{1}{2}$

$= e = \frac{1}{\sqrt{2}}$

56. Let *a, b,* and *c* be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is

1) the Geometric Mean of a and b
2) the Arithmetic Mean of a and b
3) equal to zero
4) the Harmonic Mean of a and b

Ans.(1) $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

$\Rightarrow c^2 = ab$

$\Rightarrow$ c is the G.M. of $a \& b$

57. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number then $\left[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}\right] = \left[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}\right]$ for

1) exactly one value of λ
2) no value of λ
3) exactly three values of λ
4) exactly two values of λ

Ans.(2)

$\lambda(\vec{a}+\vec{b}).[\lambda^2\vec{b} \times \lambda\vec{c}] = \vec{a}.[(\vec{b}+\vec{c}) \times \vec{b}]$

$\Rightarrow \lambda^4[\vec{a}.(\vec{b} \times \vec{c}) + \vec{b}.(\vec{b} \times \vec{c}) = \vec{a}.[\vec{b} \times \vec{b} + \vec{c} \times \vec{b}]$

$\Rightarrow \lambda^4[\vec{a}.(\vec{b} \times \vec{c})] = \vec{a}.(\vec{c} \times \vec{b})$

$\Rightarrow \lambda^4 = -1$

no real value of λ is possible

58. Let $\vec{a} = \vec{i} - \vec{k}, \vec{b} = x\vec{i} + \vec{j} + (1 = x)\vec{k}$ and $\vec{c} = y\,\vec{i} + x\vec{j} + (1 + x - y)\vec{k}.$ then $[\vec{a}, \vec{b}, \vec{c}]$ depends on

1) only y
2) only x
3) both x and y
4) neither x nor y

Ans.4) $[\vec{a}\,\vec{b}\,\vec{c}]$

$$= \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$$= [(1 + x - y) - x(1 - x)] - (x^2 - y)$$

$$= 1 + x - y - x + x^2 - x^2 + y$$

$$= 1$$

59. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

1) $\frac{2}{9}$
2) $\frac{1}{9}$
3) $\frac{8}{9}$
4) $\frac{7}{9}$

Ans.(2) Three person A, B, C

& 3 houses a, b, c

$$\Rightarrow \left(\frac{3}{3}\right) \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

60. A random variable X has Poisson distribution with mean 2. Then P(x > 1.5) equals

1) $\frac{2}{e^2}$
2) 0
3) $1 - \frac{3}{e^2}$
4) $\frac{3}{e^2}$

Ans.(3) $P(x = r) = \frac{e^{-\lambda} \lambda^r}{r!}$

$\lambda = 2$

$P(x > 1.5) = 1 - [\,p(x = 0) + P(x = 1)]$

$p(x = 0) + p\,(x = 1)$

$$= \left[\frac{e^{-2}\, 2^0}{0!} + \frac{e^{-2}\, 2^1}{1!}\right]$$

$$= e^{-2}(1 + 2) = \frac{3}{e^2}$$

$$P(x > 1.5) = 1 - \frac{3}{e^2}$$