61. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, Where $\overline{A}$ stands for complement of event A. Then events A and B are

1) equally likely and mutually exclusive
2) equally likely but not independent
3) independent but not equally likely
4) mutually exclusive and independent

Ans.(3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$P(B) = \frac{1}{3}$

$P(A \cap B) = P(A) \times P(B) = \frac{1}{4}$

62. A lizard, at an initial distance of 21 cm behind an insect, moves from rest with an acceleration of 2. cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard will catch the insect after

1) 20 s 2) 1 s 3) 21 s 4) 24 s

Ans.(3) $0.t + \frac{1}{2}.(2).t^2 = 20 \times t + 21$

$\Rightarrow t^2 - 20t - 21 = 0$

$\Rightarrow t + 21$ s.

63. Two points A and B move from rest along a straight line with constant acceleration f and f respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then

1) $(f - f')m^2 = ff'n$ 2) $(f + f')m^2 = ff'n$

3) $\frac{1}{2}(f + f')m = ff'n^2$ 4) $(f - f')n = \frac{1}{2}ff'm^2$

Ans.(4) Here $f't = f(t + m)$

$\Rightarrow t = \frac{mf}{f' - f}$ .....(1)

Also, $\frac{1}{2}f't^2 + n = \frac{1}{2}f(t + m)^2$ ..(2)

using (1) and (2)

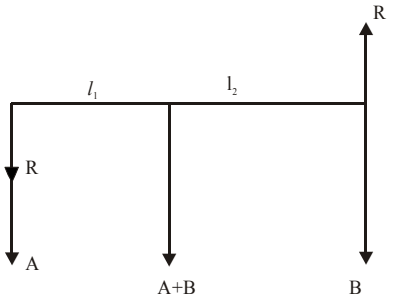
$\frac{n}{m} = \frac{1}{2}f'\left(\frac{mf}{f' - f}\right)$

$\therefore (f' - f)n = \frac{1}{2}m^2 ff'$

64. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance

a) $\frac{2H}{A - B}$ b) $\frac{H}{A + B}$ 3) $\frac{H}{2(A + B)}$ 4) $\frac{H}{A - B}$

Ans.(2)



$Al_1 = Bl_2$

$H = (l_1 + l_2)R$

Let shift distance = x

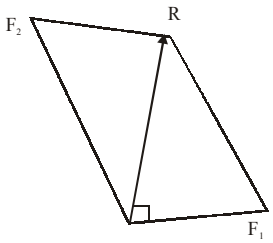
$(A+R)(l_1 - x) = (B-R)(l_2 + x)$

$$\Rightarrow x = \frac{H}{A+B}$$

65. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is :

1) 2 :1 2) $3:\sqrt{2}$ 3) 3 : 2 4) $3:2\sqrt{2}$

Ans. (4)



$R^2 + F_1^2 = F_2^2$

$$\Rightarrow \frac{F_2^2}{9} + F_1^2 = F_2^2$$

$$\Rightarrow F_1^2 = \frac{8}{9}F_2^2$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{2\sqrt{2}}{3}$$

$\Rightarrow F_2 : F_1 = 3 : 2\sqrt{2}$

66. The sum of the series $1+\frac{1}{4:2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+\cdots$ ad inf. is

1) $\frac{e-1}{\sqrt{e}}$ 2) $\frac{e+1}{\sqrt{e}}$ 3) $\frac{e-1}{2\sqrt{e}}$ 4) $\frac{e+1}{2\sqrt{e}}$

Ans.(4) $1+\frac{1}{2!}\left(\frac{1}{2}\right)^2+\frac{1}{4!}\left(\frac{1}{2}\right)^4+\frac{1}{6!}\left(\frac{1}{2}\right)+64....$

$$= \frac{e^{\frac{1}{2}} + e^{-1/2}}{2}$$

$$= \frac{\sqrt{e} + \frac{1}{\sqrt{e}}}{2}$$

$$= \frac{e+1}{2\sqrt{e}}$$

67. the value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0, is

1) $\alpha\pi$ 2) $\frac{\pi}{2}$

3) $\frac{\pi}{\alpha}$ 4) 2π

Ans.(2) $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

$\Rightarrow \int_0^{\pi} \left[\frac{\cos^2 x}{1+a^x} + a\frac{(\cos^2 x)}{a^x+1}\right] dx$

$\Rightarrow \int_0^{\pi} \frac{(1+a^x)\cos^2 x}{(1+a^x)} dx$

$\Rightarrow 2\int_0^{\pi/2} \cos^2 x\, dx$

$\Rightarrow 2\left[\frac{1}{2}\times\frac{\pi}{2}\right] = \frac{\pi}{2}$

68. The plane x + 2y - z = 4 cuts the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius

1) 3 2) 1

3) 2 4) $\sqrt{2}$

Ans.(2) Centre of the sphere $= \left(\frac{1}{2}, 0, -\frac{1}{2}\right)$

radius $= \sqrt{\frac{1}{4}+0+\frac{1}{4}+2}$

$= \frac{\sqrt{10}}{2}$

$\perp$r distance from centre to the plane

$= \left|\frac{\frac{1}{2}+0+\frac{1}{2}-4}{\sqrt{6}}\right|$

$= \frac{3}{\sqrt{6}}$

radius of the reqd. circle $= \frac{10}{4} - \frac{9}{6}$

$= \frac{30-18}{12}$

$= \frac{12}{12} = 1$

69. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then

1) $3a^2-10ab+3b^2=0$ 2) $3a^2-2ab+3b^2=0$

3) $3a^2+10ab+3b^2=0$ 4) $3a^2+2ab+3b^2=0$

Ans.(4) Here angle between the given lines is $\frac{\pi}{4}$

$$\Rightarrow 1=\left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right|$$

$$\Rightarrow 3a^2+2ab+3b^2=0$$

70. Let $x_1,x_2,...,x_n$ be n observations such that $\sum x_i^2=400$ and $\sum x_i=80$. Then a possible value of n among the following is

1) 15 2) 18 3) 9 4) 12

Ans.(2) $\sum x_i^2=400$

$\sum x_i=80$

$\sigma^2\geq 0$

$$\Rightarrow \frac{\sum x_i^2}{n}-\left(\frac{\sum x_i}{n}\right)^2\geq 0$$

$$\Rightarrow \frac{400}{n}-\frac{6400}{n^2}\geq 0$$

$\Rightarrow n\geq 16$

$\therefore$ (2)

71. A particles is projected from a 0 with velocity u at an angle of 60^0 with the horizontal. When it is moving in a direction at right angles to its direction at 0, its velocity then is given by

1) $\frac{u}{3}$ 2) $\frac{u}{2}$ 3) $2\frac{u}{3}$ 4) $\frac{u}{\sqrt{3}}$

Ans.(4) Horizontal component of velocity remains same at any point

$\therefore \quad u_x=v_x$

$u\cos 60^0=v\cos 30^0$

$$\Rightarrow v=\frac{u}{\sqrt{3}}$$

72. If both the roots of the quadratic equation $x^2-2kx+k^2+k=5=0$ are less than 5, then k lies in the interval

1) (5, 6] 2) (6, ∞) 3) $(-\infty,4)$ 4) [4, 5]

Ans.(3) $x^2-2kx+k^2+k-5=0$

let $f(x)=x^2-2kx+k^2+k-5$

since f(x) has real roots less than 5, then

D > or f(5) > 0

$\Rightarrow k^2-(k^2+k-5)>0$ or $k^2-9k+20>0$

$\Rightarrow k<5$ or $k<4$ or $k>5$

$\Rightarrow k<4$

73. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$ is equal to

1) 1 2) 0 3) 4 4) 2

Ans.(2) Since a_n, a_{n+1}, a_{n+2} are in G.P,

$a_{n+1}^2 = a_n, a_{n+2}$

$2\log a_{n+1} - \log a_n - \log a_{n+2} = 0$

Similarly $2\log a_{n+4} - \log a_{n+3} - \log a_{n+5} = 0$

$2\log a_{n+7} - \log a_{n+6} - \log a_{n+8} = 0$

Applying $c_1 + c_3 - 2c_2$,

We have $\Delta = 0$

74. A real valued function *f(x)* satisfies the functional equation *f(x - y) = f (x) f(y) - f(a - x) f (a+y)* where α is a given constant and f(0) = 1, f(2α -x) is equal to

1) *-f(x)* 2) *f(x)*

3) *f(α) + f(α -x)* 4) *f(x)*

Ans.(1) $f(a-(x-a)) = f(a)f(x-a) - f(0)f(x)$

Put x = 0, y = 0

$f(0) = f(0))^2 - [f(a)]^2$

$\Rightarrow$ $f(a) = 0$ $[\because f(0) = 1]$

From (1)

$\Rightarrow$ $f(2a - x) = f(x)$

75. If the equation $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x = 0$,

$a_1 \neq 0, n \geq 2$, ahs a positive root x = α, then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-1} + ... + a_1 = 0$ has a positive root, which is

1) greater than α 2) smaller than α

3) greater than or equal to α 4) equal to α

Ans.(2) Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x$

f(0) = 0

$f(\alpha) = 0$

$\Rightarrow f'(x) = 0$ has at least one root between $(0, \alpha)$

$\Rightarrow f'(x) = 0$ has a positive root less than α