AIEEE - 2004 Analysis
Mathematics

31. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

a) (2, 4)
b) (2, -4)
c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$
d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

- d.

From options $2y\frac{dy}{dt} = 18\frac{dx}{dt}$

$\Rightarrow y = \frac{9}{2}$

$\Rightarrow x = \frac{9}{8}$

32. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x-1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is y = 3x - 5, then the function is

a) $(x-1)^2$
b) $(x-1)^3$
c) $(x+1)^3$
d) $(x+1)^2$

- b.

From options ;
$f(x) = y = (x-1)^3$

33. The normal to the curve $x = a(1+\cos\theta)$, $y = a\sin\theta$ at 'θ' always passes through the fixed point

a) (a, 0)
b) (0, a)
c) (0, 0)
d) (a, a)

- a.

normal passes through the centre of the circle
= (a, 0)

34. If $2a+3b+6c=0,$ then at least one root of the equation $ax^2+bx+c=0$ lies in the interval

a) (0, 1)
b) (1, 2)
c) (2, 3)
d) (1, 3)

- a.

consider f(x) = $\frac{a}{3}x^3+\frac{b}{2}x^2+cx$

$f(0)=0$

f(1) = $\frac{a}{3}+\frac{b}{2}+c$

$=2c+3b+bc=0$

f(0) = f(1) = 0

$f^1(x)=ax^2+bx+c=0$

$\therefore$ By Rodlis the atleast one value of $x\in(0,1)$

35. $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}$ is

a) e
b) e - 1
c) 1 - e
d) e + 1

- b.

$\lim\limits_{n\to\infty}\sum_{r-1}^{n}\frac{1}{n}e^{r-n}=\int_0^1 e^x dx=(e-1)$

36. If $\int\frac{\sin x}{\sin(x-\alpha)}dx=Ax+B\log\sin(x-\alpha)+C$ then value of (A,B) is

a) $(\sin\alpha, \cos\alpha)$

b) $(\cos\alpha, \sin\alpha)$

c) $(-\sin\alpha, \cos\alpha)$

d) $(-\cos\alpha, \sin\alpha)$

- b.

$\int\frac{\sin x}{\sin(x-\alpha)}dx=Ax+B\log\sin(x-\alpha)+c$

Put x - α = t
dx = dt

$$\int \frac{\sin(d+t)}{\sin t} dt = \cos\alpha\ x + \sin\alpha \log \sin(x-\alpha) + c$$

$\therefore (\cos\alpha, \sin\alpha)$

37. $\int \frac{dx}{\cos x - \sin x}$ is equal to

a) $\frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + C$

b) $\frac{1}{\sqrt{2}} \log\left|\cot\left(\frac{x}{2}\right)\right| + C$

c) $\frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} - \frac{3\pi}{8}\right)\right| + C$

d) $\frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$

➢ a.

$$\int \frac{dx}{\cos x - \sin x}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x}$$

$$\frac{1}{\sqrt{2}} \int \frac{dx}{\sin \pi/4 \cos x - \cos \pi/4 \sin x}$$

$$\frac{1}{\sqrt{2}} \int \operatorname{cos}ec\left(x - \frac{\pi}{4}\right) dx$$

$$\frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + C$$

38. The value of $\int_{-2}^{3} \left|1-x^2\right| dx$ is

a) $\frac{28}{3}$

b) $\frac{14}{3}$

c) $\frac{7}{3}$

d) $\frac{1}{3}$

➢ a.

$\int_{-2}^{-1}(x^2-1)\,dx + \int_{-1}^{1}(1-x^2)dx + \int_{1}^{3}(x^2-1)dx$

$= \left[\frac{x^3}{3}-x\right]_{-2}^{-1} + \left[x-\frac{x^3}{3}\right]_{-1}^{1} + \left[\frac{x^3}{3}-x\right]_{1}^{3}$

$= \frac{28}{3}$

39. The value of $I = \int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}} dx$ is

a) 0
b) 1
c) 2
d) 3

➢ c.

$I = \int_0^{\pi/2} \sin x + \cos x$ $\quad [\because \sqrt{1+\sin 2x} = \sin x + \cos x]$

$= [-\cos x + \sin x]_0^{\pi/2}$
= 2

40. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\pi/2} f(\sin x) dx$, then A is

a) 0
b) π
c) $\frac{\pi}{4}$
d) 2π

➢ b.

$I = \int_0^{\pi} xf(\sin x)dx$

$I = \int_0^{\pi} (\pi - x)\, f(\sin x)\, dx$

$2I = \pi \int_0^{\pi} f(\sin x)$

$2I = 2\pi \int_0^{\pi/2} f(\sin x)$

$I = \pi \int_0^{\pi/2} f(\sin x)$

$\therefore A = \pi$

41. If $f(x) = \dfrac{e^x}{1+e^x}$, $I_1 \int_{f(-a)}^{f(a)} xg\{x(1-x)dx\}$ and $I_2 = \int_{f(-a)}^{f(a)} g[x(1-x)\}dx,$ then the value of $\dfrac{I_2}{I_1}$ is

a) 2
b) -3
c) -1
d) 1

➢ a.

$I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)dx$

$I_1 = \int_{f(-a)}^{f(a)} (f(a)+f(-a)-x)g\,[(f(a)+f(-a)-x)\{1-f(a)+f(-a)-x\}$

$= \int_{f(-a)}^{f(a)} (1-x)g\,[(1-x)x]dx \;\; \{\because f(a)+f(-a)=1\}$

$= I_2 - I_1 \Rightarrow 2I_1 = I_2 \Rightarrow \dfrac{I_2}{I_1} = 2$

42. The area of the region bounded by the curves $y = |x-2|,\ x=1,\ x=3$ and the x-axis is

a) 1
b) 2
c) 3
d) 4

➢ a.

$\frac{1}{2}(1)(1)+\frac{1}{2}(1.1)=1$

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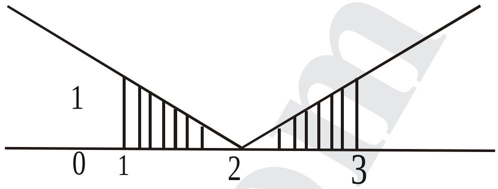
Area = $\int_{1}^{2} 2-x+\int_{2}^{3} x-2 \ =1$



43. The differential equation for the family of curves $x^2+y^2-2ay=0$, where a is an arbitrary constant is

a) $2(x^2-y^2)y'=xy$,

b) $2(x^2+y^2)y'=xy$

c) $(x^2-y^2)y'=2xy$

d) $(x^2+y^2)y'=2xy$

➢ c.

a = $\frac{x^2+x^2}{2y}$

$2x+2yy_1-2-y_1=0$

$\Rightarrow a=\frac{x+yy_1}{y_1}$

$x^2+y^2-2\,xy\left(\frac{x+yy_1}{y_1}\right)=0$

$(x^2-y^2)y'=2xy$

44. The solution of the differential equation $ydx+(x+x^2y)dy=0$ is

a) $-\frac{1}{xy}=C$

b) $-\frac{1}{xy}+\log y=C$

c) $\frac{1}{xy}+\log y=C$

d) log y = Cx

➢ b.

y.dx + $(x+x^2y)dy=0$

$\Rightarrow y.\frac{dx}{dy}+x+x^2y=0$

$\Rightarrow \frac{dx}{dy}+\frac{x}{y}=-x^2$

$\Rightarrow -\frac{1}{x^2}\frac{dx}{dy} - \frac{1}{xy} = 1$

$\Rightarrow$ Put $\frac{1}{x} = t \Rightarrow -\frac{1}{x^2}\frac{dx}{dy} = \frac{dt}{dy}$

$\Rightarrow \frac{dt}{dy} - \frac{1}{y}t = 1$

$I.F = \frac{1}{y}$

$\therefore$ The solution is $t.I.F = \int 1.I.F\ dy$

$\Rightarrow t.\frac{1}{y} = \int 1.\frac{1}{y}dy$

$\frac{1}{xy} = \log y + c$

45. Let A(2, -3) and B(-2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line 2x + 3y = 1, then the locus of the vertex C is the line
a) 2x + 3y = 9
b) 2x - 3y = 7
c) 3x + 2y = 5
d) 3x - 2y = 3

➢ a.

Let (h, k) he C

centroid G = $\left(\frac{h}{3}, \frac{k-2}{3}\right)$

$\left(\frac{h}{3}, \frac{k-2}{3}\right)$ lies in 2x + 3x = 1

$\Rightarrow 2h + 3k = 9$
$\Rightarrow 2x + 3y = 9$