AIEEE - 2004 Analysis

Mathematics

61. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by

a) 40
b) 30
c) 25
d) 15

➢ a.

$\vec{F}_{net}=\vec{F}_1+\vec{F}_2$

$=7\hat{i}+2\hat{j}-4\hat{k}$

$\bar{s}=4\hat{i}+2\hat{j}-2\hat{k}$

Wd = $\vec{F}.\vec{S}$
= 28 + 4 + 8
= 40

62. If $\bar{a},\bar{b},\bar{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\bar{a}+2\bar{b}+3\bar{c}$, $\lambda\bar{b}+4\bar{c}$ and $(2\lambda-1)\bar{c}$ are non-coplanar for

a) all values of λ
b) all except one value of λ
c) all except two values of λ
d) no value of λ

➢ c.

They are coplaner is

$$\begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & 2\lambda-1\end{vmatrix}=0$$

$=\lambda(2\lambda-1)-2(0)+3(0)=0$

if $\lambda=0$ and $\lambda=\frac{1}{2}$

63. Let $\bar{u},\bar{v},\bar{w}$ be such that $|\bar{u}|=1, |\bar{v}|=2, |\bar{w}|=3.$ If the projection $\bar{v}$ along $\bar{u}$ is equal to that of $\bar{w}$ along $\bar{u}$ and $\bar{v},\bar{w}$ are perpendicular to each other then $|\bar{u}-\bar{v}+\bar{w}|$ equals

a) 2
b) $\sqrt{7}$
c) $\sqrt{14}$
d) 14

➢ c.

Given $\frac{\vec{v}.\vec{u}}{|u|} = \frac{\vec{w}.\vec{u}}{|u|}$

and $\vec{v}.\vec{w} = 0$

Now let $y = |\vec{u} - \vec{v} + \vec{w}|$

$y^2 = u^2 + v^2 + w^2 - 2u.v - 2v.w + 2u.w$

= 1+ 4+ 9- 2u.v - 0 + 2u.v

= 14

y = $\sqrt{14}$

64. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be non-zero vectors such that $(\bar{a} \times \bar{b}) \times \bar{c} = \frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$. If θ is the acute angle between the vectors $\bar{b}$ and $\bar{c}$, then $\sin\theta$ equals

a) $\frac{1}{3}$

b) $\frac{\sqrt{2}}{3}$

c) $\frac{2}{3}$

d) $\frac{2\sqrt{2}}{3}$

➢ d.

$(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} = |\vec{b}||\vec{c}|\left|\cos\theta + \frac{1}{3}\right|\vec{a}$

$\Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$

65. Consider the following statements :
1) Mode can be computed from histogram
2) Median is not independent of change of scale
3) Variance is independent of change of origin and scale.
Which of these is/are correct
a) only (1)
b) only (2)
c) only (1) and (2)
d) (1), (2), and (3)

➢ c.

66. In a series of 2 n observations, half of them equal a and remaining half equal -*a*. If the standard deviation of the observations is 2, then $|a|$ equals

a) $\frac{1}{n}$

b) $\sqrt{2}$

c) 2

d) $\frac{\sqrt{2}}{n}$

➢ b.

$$S.D = \sqrt{\frac{(x-\bar{x})^2}{x}}$$

$$\bar{x} = \frac{na - na}{2x} = 0$$

$$\text{S.D} = \sqrt{\frac{x^2 + a^2 + a^2 + ...a\ 2x\ times}{2x}}$$

$$\Rightarrow 2 = \sqrt{\frac{2a^2x}{2x}}$$

$$\Rightarrow \sqrt{a^2} = 2$$

$$\Rightarrow |\ a\ | = \sqrt{2}$$

67. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is

a) $\frac{3}{20}$

b) $\frac{1}{5}$

c) $\frac{7}{20}$

d) $\frac{4}{5}$

➢ c.

$$P(A) = \frac{4}{5},\quad P\bar{A} = \frac{1}{5}$$

$$P(B) = \frac{3}{4},\ P\bar{B} = \frac{1}{4}$$

then $P(A)P(\bar{B}) + P(\bar{A})\ P(B)$

$\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}=\frac{7}{20}$

68. A random variable X has the probability distribution :

X :	1	2	3	4	5	6	7	8
$p(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number } and F = { X < 4}, the probability $P(E\cup F)$ is :

a) 0.87
b) 0.77
c) 0.35
d) 0.50

➢ b.

P $(E\cup F)=P(E)+P(F)-P(E\cap F)$
P(E) = 0.12 + 0.23 + 0.20 + 0.20 + 0.07 = 0.62
P(F) = 0.15 + 0.23 + 0.12 = 0.50
$P(E\cap F)=0.23+0.12=0.35$
$P(E\cup F)=0.62+0.50-0.35=0.77$

69. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is :

a) $\frac{37}{256}$

b) $\frac{219}{256}$

c) $\frac{128}{256}$

d) $\frac{28}{256}$

➢ d.

mean $\Rightarrow np=4$

variance $\Rightarrow npq=2$

4q = 2 $q=\frac{1}{2}$ $p=\frac{1}{2}$

then n = 8
Then the probability of 2 successes

$^8c_2\left(\frac{1}{2}\right)^6,\left(\frac{1}{2}\right)^2=\frac{28}{256}$

70. With two forces acting at a point, the maximum effect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are

a) $\left(2+\sqrt{2}\right)N$ and $\left(2-\sqrt{2}\right)N$

b) $\left(2+\sqrt{3}\right)N$ and $\left(2-\sqrt{3}\right)N$

c) $\left(2+\frac{1}{2}\sqrt{2}\right)N$ and $\left(2-\frac{1}{2}\sqrt{2}\right)N$

d) $\left(2+\frac{1}{2}\sqrt{3}\right)N$ and $\left(2-\frac{1}{2}\sqrt{3}\right)N$

➢ c.

$R=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

given $F_1+F_2=4$ ...(1)

$\& F_1^2+F_2^2=9$ ...(2)

solving (1) & (2)

we get $F_1=2+\frac{\sqrt{2}}{2}$

$F_2=2-\frac{\sqrt{2}}{2}$

71. In a right angle ΔABC, $\angle A=90°$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is

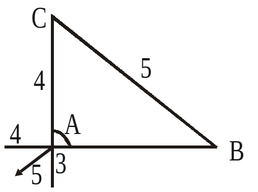
a) 3
b) 4
c) 5
d) 9

➢ c.

Clearly $\vec{F}$ acts at pt A
and its components perpendicular to

AC & AB are $\frac{16}{4}(=4)\ \&\left(\frac{9}{3}\right)=(3)$

$\therefore|\vec{F}|=\sqrt{3^2+4^2}=5$ $\{\because \text{Ac} \,\&\, \text{AB are} \perp^{r}\}$



72. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC, are in equilibrium. Then $\vec{P} : \vec{Q} : \vec{R}$ is

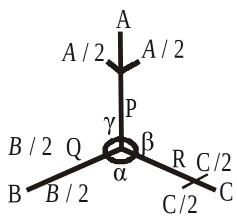
a) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$

b) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$

c) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$

d) $\text{cosec}\frac{A}{2} : \text{cosec}\frac{B}{2} : \text{cosec}\frac{C}{2}$

➢ a.



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\Rightarrow \frac{R}{\sin\left|180° - \frac{B+C}{2}\right|} = \frac{Q}{\sin\left|180° - \frac{A+C}{2}\right|} = \frac{R}{\sin\left|180° - \frac{A+B}{2}\right|}$$

$$\Rightarrow \frac{P}{\sin\left|\frac{B+C}{2}\right|} = \frac{Q}{\sin\left|\frac{A+C}{2}\right|} = \frac{R}{\sin\left|\frac{A+B}{2}\right|} \Rightarrow \frac{P}{\cos\frac{A}{2}} = \frac{Q}{\cos\frac{B}{2}} = \frac{R}{\cos\frac{C}{2}}$$

73. A particle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5 km/h. If AB = 12 km and BC = 5 km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively

a) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h

b) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h

c) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h

d) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h

➢ a.

$\text{Average speed} = \dfrac{\text{Total distance}}{\text{Total time}}$

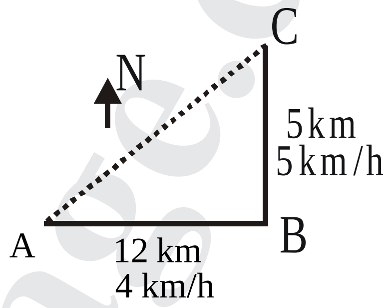
$= \dfrac{12+5}{\dfrac{12}{4}+\dfrac{5}{5}}$

$= \dfrac{17}{4}\, km/h$

$\text{Averge velocity} = \dfrac{\text{Total Displacement}}{\text{Total time}}$

$= \dfrac{\sqrt{12^2+5^2}}{\dfrac{12}{4}+\dfrac{5}{5}}$

$= \dfrac{13}{4}\,\text{km/h.from A to C}$



74. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is

a) $\frac{1}{8}$ m/s

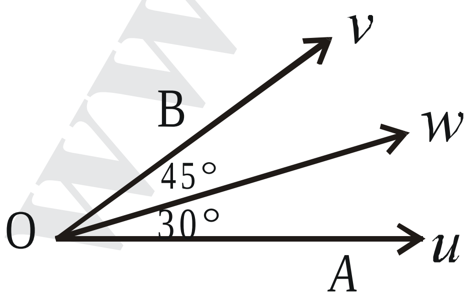
b) $\frac{1}{4}(\sqrt{3}-1)$ m/s

c) $\frac{1}{4}$ m/s

d) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s

➢ d.

$$u = \frac{1/4\sin 30^\circ}{\sin 75^\circ} = \frac{\frac{1}{4}\times\frac{1}{2}}{\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\frac{1}{2}} = \frac{1}{8}\left|\sqrt{6}-\sqrt{2}\right| m/s$$



75. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to :

a) u^2 / g

b) $4u^2 / g^2$

c) $u^2 / 2g$

d) 1

➢ b.

The only distinct case will be when angles of projection are $\theta \,\&\, 90 - \theta$

Now $t = \frac{2u\sin\theta}{g}$

$$\therefore t_1^2 + t_2^2 = \left(\frac{2u\sin\theta}{g}\right)^2 + \left(\frac{2u\cos\theta}{g}\right)^2$$

$$= \frac{4u^2}{g^2}$$